FROM DOING TO UNDERSTANDING: AN ASSESSMENT OF MALAYSIAN PRIMARY PUPILS' NUMBER SENSE WITH RESPECT TO MULTIPLICATION AND DIVISION

Munirah Ghazali
Sharifah Norhaidah Idros
School of Educational Studies, University Science Malaysia
and
Alistair McIntosh
Faculty of Education, University of Tasmania, Australia

Number sense, an integral part of the curriculum in primary school mathematics, has always been thought of as playing a major role in pupils' understanding and use of numbers in calculation. Research in number sense has been carried out in the West, as well as in Malaysia. That a study of such a nature be done to reflect on the abilities of primary school pupils in this area, the authors embarked on the development of an instrument to assess pupils' number sense. The instrument constructed focused on counting, addition and subtraction, multiplication and division, and place value, but this study examined pupils' abilities in multiplication and division. The responses obtained after administering the instrument to 128 pupils were analysed and the authors found the instrument to be suitable for measuring the acquisition of number sense with respect to multiplication and division.

INTRODUCTION

Many mathematics educators worldwide as well as in Malaysia are concerned whether primary school students demonstrate understanding of numbers or whether they just apply standard algorithms to calculations, which could have easily been computed with calculators (Leutzinger & Bertheau, 1989; Burns, 1989, Munirah 2000). Yang (1995) suggested that this could be due primarily to the mindless application of the standard written algorithms that students learned in school. Students are good rule followers but unfortunately do not always understand the procedures they learned (Hiebert & Wearne, 1986). They are adept at manipulating and following symbol rule but are less able at making sense of numerical situations. Moreover, while emphasis on computational skills may produce high computational scores, the extent to which these processes transfer to the students' understanding is unknown. Curricular reform documents (such as National Council of Teacher of Mathematics, 1989; Cockcroft, 1982) emphasize the importance of number sense based on the rationale that number sense will be very helpful to understand numbers in general. A number of mathematics educators seem to agree that the difficulties experienced by children in solving mathematics exercises are closely related to the development of number sense thinking (Leutzinger & Bertheau, 1989; Burns, 1989). Although considerable attention to number sense is occurring in countries like the US, Australia and the UK, the term 'number sense' is rarely heard in mathematics education, national mathematics curriculum, school classrooms, teachers or educational journals in Malaysia. Even though many good teachers are undoubtedly teaching mathematics in ways that lead their students to develop good understanding in numbers and operations, the relationships between numbers and operations, and numbers and computations, the researchers believe that the development of number sense will play an important role in elementary mathematics education in Malaysia.

Recent research on students' number sense in Malaysia showed that there were students who could perform the arithmetic calculation well but lacked number sense (Munirah, 2000). Moreover, analyses from the study showed that while students were able to do calculations for certain computation questions ironically they face difficulty doing the same questions when posed in a number sense format. The research on number sense so far has established that the unchallenged belief that 'if students can compute then they have understood' is now being questioned and rightly so seeing that The Curriculum and Evaluation Standards for School Mathematics (1989) states that "students must understand numbers if they are to make sense of the ways numbers are used in their everyday world". Students can compute but have different mental representation of fractions. The same mathematics scenario is apparent in Malaysian schools. Research in Malaysia (Munirah & Noor Azlan, 1999) has strongly indicated that most school children do not display good number sense and predictably have an average standing in mathematics achievement compared to other nations. An analysis of the Third International Mathematics and Science Study – Repeat (TIMSS-R) showed that Malaysians students perform well above the international average for questions that require computation but face difficulty and thus perform below the international average for questions that require understanding of basic concepts (Kementerian Pendidikan Malaysia, 1989).

FRAMEWORK TO ASSESS CHILDREN'S NUMBER SENSE IN MULTIPLICATION AND DIVISION

Number sense is difficult to define (Hope, 1989; Sowder and Schappelle, 1989; Greeno, 1991; Case, 1998) and may mean different things in different contexts but a situation where a person displays number sense could be identified (Greeno, 1991). Moreover, situations where students display a lack of number sense could be identified too. (Hope, 1989). Apart from being difficult to define, number sense is also difficult to measure (Sowder and Schappelle, 1989). Nonetheless, despite being difficult to define and measure, number sense is an important trait for students to have (Hope, 1989). Greeno (1991) suggests that number sense is a cognitive skill as a product of learning and not as an objective of teaching. When highlighting number sense, students should focus on their solution strategies rather than on a "right answer," on thinking rather than on the mechanical application of rules, and on student-generated solutions rather than on teacher-supplied answers. It is the authors' belief that the study and development of such abilities in pupils should be an integral part of the mathematics curriculum and that the early years of schooling are deemed crucial in providing the kind of positive start to students' number sense learning. Thus, a research project was undertaken with the aim of developing an instrument to assess pupils' number sense and developing a framework in the four components: counting, addition and subtraction, multiplication and division and place value.

METHODOLOGY

Development of the Instrument

The development of the instrument to assess students' number sense in the components of counting, addition and subtraction, multiplication and division and place value went through a cyclical process of construction, small-group testing, refinement and fine-tuning until the finished instrument was finally tried out on a target sample of 128 primary school pupils.

Initially, an extensive review of literature related to number sense and numeracy such as "Count Me In" from Australia, "Leverhume Numeracy Project" carried out in the UK, "Supporting Literacy and Numeracy" in Australia, was done and a comparison of those findings applied to the current situation in Malaysian primary school mathematics. A draft model of the instrument was put together with input contributed by primary school teachers, experts from the university and officers from the Center of Curriculum Development as well as from the Education and Research Planning Unit of the Ministry of Education, Malaysia in a two-day workshop. This draft model was tried out on a few schoolchildren so that mistakes were rectified and the model further improved.

This was followed by a second workshop to further refine the first draft. An international consultant from the University of Tasmania was invited to help with this process having done extensive work on a similar study in Tasmania and Australia. His guidance proved invaluable for there were some items constructed that were not necessary and several other items that were restructured so as to make it more powerful in eliciting understanding of number sense from school children. Following this, the instrument was improved further to take into account the suggestions and new inputs received during the workshop. All in all, this instrument underwent a series of try-outs and amendments until the group felt that it had undergone a rigorous cycle of rectifications and fine-tuning for it to be considered worthy of being piloted to a small group.

The instrument to categorize pupils' abilities on school mathematics had been drawn up by the primary school teachers during the first workshop. The items constructed for this purpose followed the current standard mathematical curriculum and the categorization of pupils into groups of differing mathematical abilities (excellent, good and fair) were also based on standards practiced in Malaysian schools. The interviews were all videotaped so that the playback could be used to glean for unexpected behaviour from the pupils as well as how the interview process could be further improved. Much valuable information was extracted from the playback for it visibly showed some computational strategies exhibited by school children that could otherwise be missed altogether.

Following this, a series of meetings and discussions were done to improve on the instruments as well as on the interviewing process and finally the finished instrument carried out on a larger sample of 128 schoolchildren from Years 1, 2 and 3.

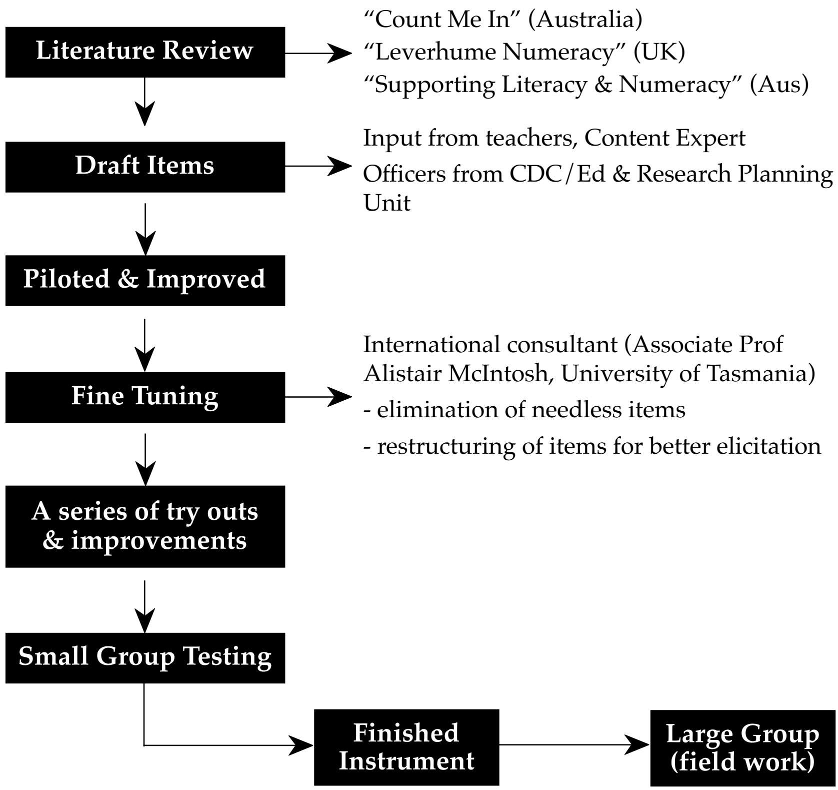

Figure 1: A summary of the process for developing the number sense instrument

THE RESEARCH INSTRUMENT

The final version of the instrument was actually made up of four separate smaller ones produced specifically for the four components namely counting, addition and subtraction, multiplication and division and place value. The items in each separate component were constructed to fulfill three main numerical representations that is contextual, pictorial/objects and symbolic. This paper will only focus on the multiplication and division component and discussion as the other components have been discussed elsewhere (Munirah, Shafia, Sharifah, Zurida, Fatimah (2003). The items for multiplication and division included word problems, mental computation, written computation and creating problems from a given written computation.

For the multiplication and division component, there were three categories of questions namely *contextual*, *pictorial* and *symbolic*. The *contextual* category comprised word problems in the form of multiplication and addition stories. There were two multiplication and two division questions in this category and the questions were arranged from easy to difficult.

Pupils were asked to approach the questions in the instrument in two ways. First, students were asked to do the question mentally without using paper and pencil. After the pupils had given the answer, the researchers asked the students to explain the strategy that was used in obtaining the answer. Next, the pupil was asked to give a written answer to the question and asked to explain how the strategy worked. The pictorial category was made up of multiplication and division questions based on a given picture. There were six multiplication and division questions. The questions required pupils to state the total number of objects in a picture and to write the mathematical sentence involving multiplication and division. For example, the researchers gave the pupils a picture of six hens with their legs hidden and asked the pupils to tell the total number of hens' legs. Pupils were required to give the answer and also to explain their strategy to arrive at the given answer. For this item, a beginner's response would be categorized as " guess, wrong", "wrong and point with finger", "wrong and count using finger". Responses "correct but hesitant", "correct, count in twos but hesitant", "correct and count in fours but hesitant" and "correct but count with fingers, hesitant" are categorized as emergent responses. Responses that were categorized as competent are "correct and count 1-1", "efficient, count in two's", "correct, count in fours", "subitemise", "correct, point with finger" and "correct multiply efficiently".

The symbolic category consisted of multiplication and division questions in the form of mathematical sentences. There were four questions that require the pupils to visualize the situation and create stories based on the given mathematical sentence. Two of the items actually required the pupils to state the reason and one question requires the pupil to give a different mathematical sentence that would return the same answer as the given mathematical sentence. For this category, the pupils were asked to explain their strategies used to solve the given questions.

ANALYSIS OF DATA

The responses of the 128 pupils to each of the components were analyzed using BIGSTEPS (a Rasch Measurement program for obtaining objective, fundamental measures from stochastic observations of ordered category responses) and FACTOR ANALYSIS, a data reduction module using SPSS for Windows Release 11.0. The Rasch analysis (Partial Credit Model) was carried out to determine whether the multiplication and division items were working together to form a unidimensional scale of "multiplication and division" to measure which of the items are more difficult ..Item response modelling, and specifically the Rasch model, uses the interaction between persons and items to determine the probability of success of each person on each item of a test. This provides a set of scores that describes the locations of persons and items along an underlying variable (Griffin 1997). In the study reported here the underlying variable is multiplication and division competence and the positions of items on the scale reflect increasing competence. The model (Rasch 1980) can be represented by the equation:

$$P\left\{\frac{x}{\beta_v},\ \delta_i\right\}=\frac{e^{x(\beta_v - \delta_i)}}{1+e^{x(\beta_v - \delta_i)}}$$

where P is the probability of a result (x) on an item (i) attempted by a person (v). In the case of dichotomous (right/wrong) responses, as in this study, x can take the value 0 or 1.

β_v is the position of person v on the variable and is referred to as the ability parameter.

δ_i is the position of the item i on the variable and is referred to as the difficulty parameter.

Validation of Items Used To Assess Multiplication and Division of Number Sense in Years 1, 2 and 3

The validation of an instrument refers to the ability of that instrument to measure the particular construct that it set out to do. In this study, the instrument developed was to assess understandings in 'multiplication and division' of number sense in pupils of Years 1, 2 and 3, namely:

i. The use of pictorial representations in solving problems in multiplication and division.

ii. To write mathematical sentences representing multiplication and division.

iii. To solve problems in multiplication and division.

iv. To represent situations of multiplication division by involving mathematics sentences in multiplication and division.

Factor analysis on the 18 items was done to ascertain if there were clear factors for number sense in multiplication and division. Five factors were extracted but the lone item in Factor 5 i.e. Item 1 was transferred to Factor 4 (refer to Figure 2). Naming of the factors was done so as to capture the essence of the items in each factor.

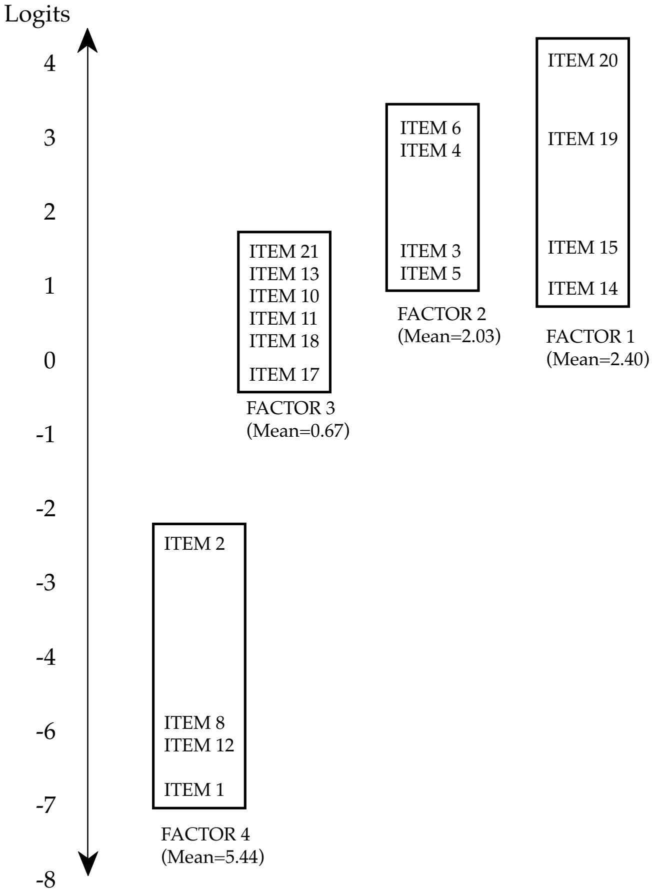

Figure 1: Mapping of the 18 items defined by 4 factors in multiplication and division component

The final four factors represented these concepts namely (1) The use of pictorial representations in solving problems in multiplication and division, (2) To write mathematical sentences representing multiplications division, (3) To solve problems in multiplication and division and (4) To represent situations and to create stories based on multiplication and division operations.

The factors were sequenced according to the mean values. Low mean values indicated that pupils could easily answer correctly on these items as compared to factors having higher mean values. As can be seen from figure 7, Factor 4 which consists of items involving the use of pictorial representations in solving problems in multiplication and division were the easiest to be done indicated by a mean value of -5.44 and SD of 0.80, followed by Factor 3 consisting of items involving writing mathematical sentences representing multiplication and division with a mean value of 0.67, SD of 0.37, then Factor 2 representing items to do with solving problems in multiplication and division (mean=2.03, SD=0.43) and finally Factor 1 which consists of items involving representing situations and creating stories based on multiplication and division operations (mean=2.40, SD=0.52).

The sequencing of factors on Table 7 follows the hierarchical mastery of the various skills needed to achieve the Factor at the top end. In other words, pupils needed to firstly use pictorial representations in solving problems in multiplication and division, before they can write mathematical sentences representing multiplication and division. When these skills are in place then they can begin to solve problems in multiplication and division and using these skills as enablers they now represent situations and create stories based on multiplication and division operations.

There was a clear jump in terms of mean values between Factor 4 and Factor 3 along the continuum. This could be due to the fact that the use of pictorial representations in solving problems in multiplication and division (Factor 4) involved only competent counting. Pupils who were counting "one-by-one" were also categorised as competent. This activity does not show pupils' understanding in multiplication and division. What should have been done was to place counting in multiples of twos, threes and fours as competent and counting one-by-one as being at the 'beginning' level. As such it was felt that Factor 4 be taken out when assessing number sense in multiplication and division. Nevertheless, there was also difficulty in differentiating the other three factors exclusively. Even though all the three factors were needed to evaluate the component of multiplication and division, they did not seem to be independent, except for Factor 4.

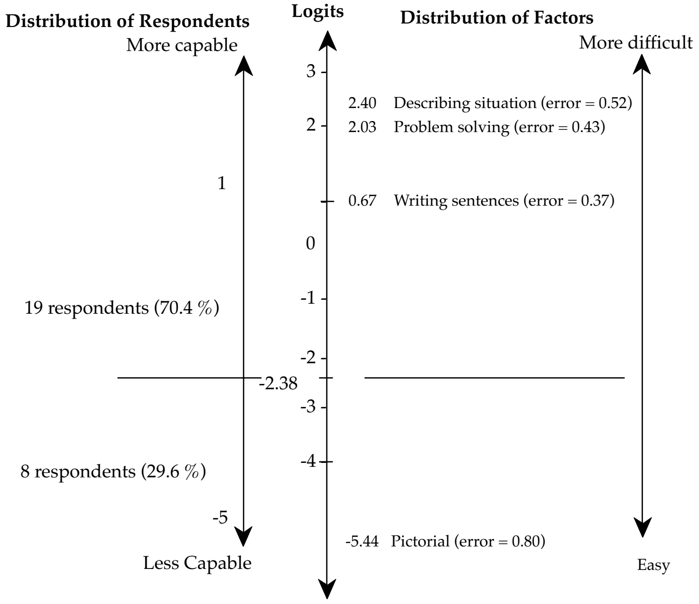

Figure 3: Means and separation of two standard errors for number sense in multiplication and division

The four factors extracted could be further collapsed to form two big groups that are statistically different between them. The first group consist of the items in Factor 4 (mean =-5.44) making up the skills easiest to acquire by pupils (ie. the use of pictorial representations in solving problems in multiplication and division) while the second group consist of Factors 3 (mean= 0.67), 2 (mean=2.03) and 1 (mean=2.40) namely those skills needed to write mathematical sentences representing multiplication and division, to solve problems in multiplication and division and to represent situations and creating stories based on multiplication and division operations. This delineation is determined by analysing the mean values as well as the value of "2" times the SD around the mean values as shown in Figure 8.

Even though the person's separation allows the formation of three groups (separation=3.05) in this study, the ability of candidates were grouped into those that have acquired the competencies for multiplication and division and those that have not yet acquired these same competencies based on the factor groupings. 29.6% of the pupils were located in the factor grouping of not having acquired the competencies in multiplication and division. They were found to be still needing the concrete representations in order to solve problems in multiplication and division.

70.4 % of the pupils fell into the factor grouping of already having acquired multiplication and division to solve mathematical problems by generating sentences, engaging in problem solving as well as representing situations and creating stories based on multiplication and division operations. These candidates no longer needed concrete representations in order to solve problems involving multiplication and division.

There was a wide gap (of more than 1 logit) between the skills needed to use pictorial representations and the skills needed to generate mathematical sentences in the process of solving problems in multiplication and division. The analysis showed that the test items were able to measure the construct of number sense in multiplication and division as suggested. It also showed that the items are capable of functioning together to form a unidimensional scale measuring a single construct. Due to this, the items can be said to have been validated and to have reliability.

The Difficulty of Items Used in Assessing Number Sense Understanding in the component of Multiplication and Division

Eighteen items that had been analysed to be appropriate and valid for use in the test have differing levels of difficulty that were distributed in a normal fashion ranging from having a value of 4.0 logits (error=0.82) for the most difficult item (Item 20) to the least difficult item (Item 1) having logit value of –7.12 (error=1.16). These have been summarised in Table 1. The respondents were also found to have been distributed in a normal fashion with the person's measure ranging from 3.12 logits (error=0.51) for the respondent displaying highest ability to a value of –6.34 logits (error=0.73) for the respondent with the lowest ability. Table 1 summarises this information.

Table 1
Statistics Obtained For Item Measures

Item	Measure
20	4.00
19	2.90
6	2.90
4	2.55
15	1.62
5	1.34
3	1.34
14	1.08
21	1.08
13	1.08
10	0.83
11	0.61
18	0.40
17	-0.01
2	-2.72
8	-5.95
12	-5.95
1	-7.12

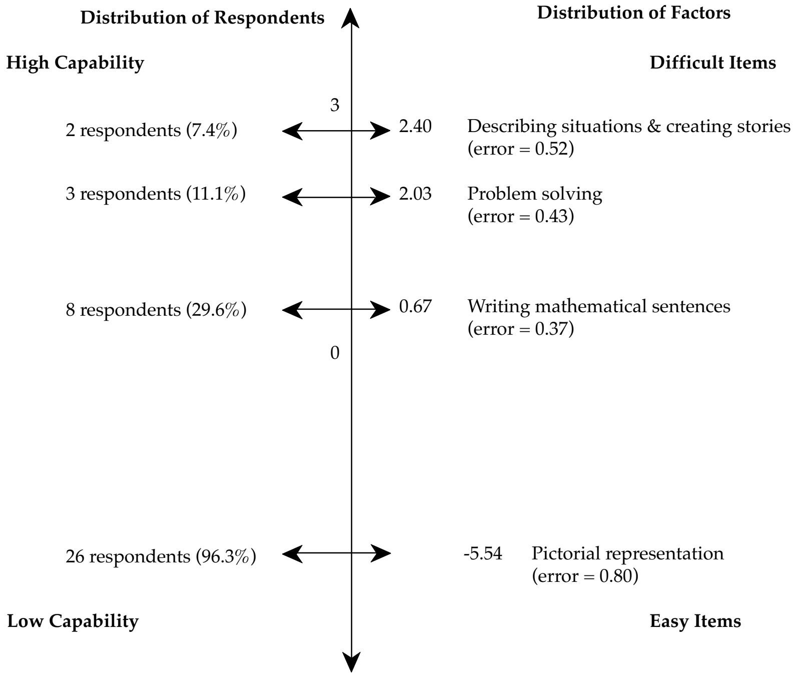

Figure 4: Mapping of Candidates To Items for Multiplication and Division

Figure 4 shows the pupils distribution regarding their abilities in answering the items at different levels of difficulties concerning number sense in the component of multiplication and division. Only 7.4 % (two pupils) of pupils were successful in answering correctly items in Factor 1 i.e. to represent situations and creating stories based on multiplication and division operations compared to 92.6% who failed to get it right. Only 11.1% of pupils were successful in Factor 2 i.e. to solve problems in multiplication and division followed by 29.6% successful on items in Factor 3 i.e. to write mathematical sentences representing multiplication and division. The majority of pupils, 96.3% could answer items in Factor 4 i.e. in the use of pictorial representations in solving problems in multiplication and division. This meant that 3.7% of respondents were not able to answer correctly any of the items from all 4 factors. In other words, the 3.7% (I pupil) of the respondents could be said to have not acquired understanding the operations and number sense of multiplication and division.

CONCLUSIONS

From the 21 items initially used, only 18 items were found to be appropriate to measure number sense understanding in multiplication and division of pupils in Years 1, 2, and 3. All 18 items fitted the Rasch measurement model. The items created a unidimensional construct which was the ability to measure for the acquisition of number sense understanding in multiplication and division. The construct is composed of 4 factors arranged in a hierarchical manner with Factor 4 (The use of pictorial representations in solving problems in multiplication and division) as the simplest subscale where the majority of pupils were found to have displayed the skills and strategies, followed by Factor 3 (To write mathematical sentences representing multiplication and division) then by Factor 2 (To solve problems in multiplication and division) and lastly by Factor 1 (To represent situations and creating stories based on multiplication and division operations). Factor 1 being at the top end of the unidimensional scale indicates that it is the subscale where only a few have displayed success (7.4%).

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APPENDICES

Question MDQ1

Instruction

Teacher show picture MDP1 to the student.

Teacher asks these questions

How many hens are there in the picture?
{Student Answer: ______________________}
Can you explain how you got the answer?

Student's Response		Category (Please tick)	Remarks
No answer	1		
Wrong, guess	2		
Wrong, point with finger	3	Beginning	
Wrong, count with finger	4		
	5		
Correct, count in 1's, hesitant	6		
Correct, count in 2's hesitant	7		
Correct, count in 3's, hesitant	8	Emergent	
Correct, point with finger, hesitant	9		
	1		
Correct, count in 1's, fluent			
Correct, count in 2's fluent			
Correct, count in 3's fluent		Competent	
Correct, fluent (subitemise)	1		
Correct, point with finger, fluent			